LandStar8 FAQ Series

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Using Least Square Analysis to Compute the Relative Accuracy of Surveyed Vectors

The vector lengths and bearings for boundaries can be uniquely computed from the coordinates of the boundary corners. However, there is a danger when using single measurements to determine parcel coordinates.

Optical measurements might include blunders from incorrect prism offsets, instable setups, incorrect backsights, bad meteorological observations and incorrect compensators. GNSS coordinates could be based on a bad or noisy fix, which is exasperated when using GPS under canopy; or with long correction baselines or noisy VRS network solutions.

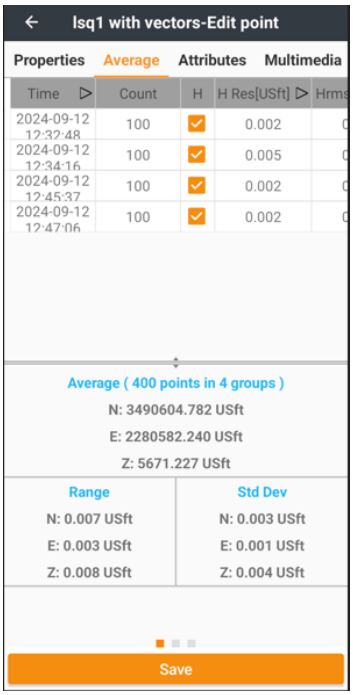
For this reason, in LandStar, you are encouraged to use the Verified survey method to make redundant GNSS observations for any important measurement. The Verified survey method records groups of single epoch measurements. Typically, these groups will have 100 epochs at 5 Hz, with full GNSS dumps followed by a 30-second wait after the engine refixes with a completely new solution. These forced independent measurements help guarantee that only valid, repeatable fixes are used:

> While it is probable that a receiver may get two bad fixes in a row, it is very-very improbable that two bad fixes will ever match.

A full receiver engine reset, coupled with the delay in satellite acquisition and an additional wait after fix, help ensure a significant constellation change as satellites move past the local canopy obstructions.

The more independent measurements we make at each corner, the more confidence we have in detecting bad measurements. For example, the four measurements of this corner have a horizontal

range of less than 0.01 feet:

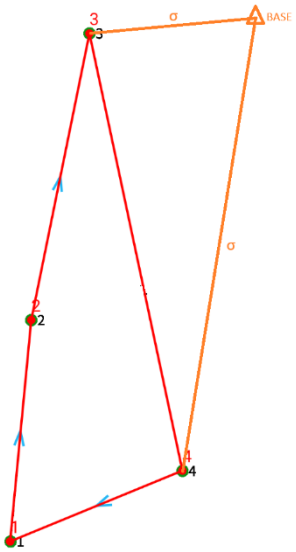


It is improbable that any of these fixes are ‘bad fixes’ as they were collected over a 15-minute period and were separated by complete GNSS engine resets.

Additional confidence can be obtained by using two or more bases for each measurement. Because the least squares analysis considers the ECEF (Earth Centered Earth Fixed) vectors between multiple bases, not the coordinates, exactly matching the coordinate solutions between multiple bases is not required as only the vector lengths are used.

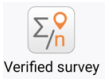
LandStar utilizes a constrained least-squares adjustment holding the base point(s) as control points. Because direct GNSS measurement of parcel corners are not made directly with a Base on the first corner and a Rover on the second corner, the error estimate between corners (the parcel lines) is a function of the estimated vector error from the first corner to the Base and then from the Base to

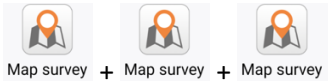
the second corner:



Collecting Measurements

LandStar8 accepts these types of redundant measurements:

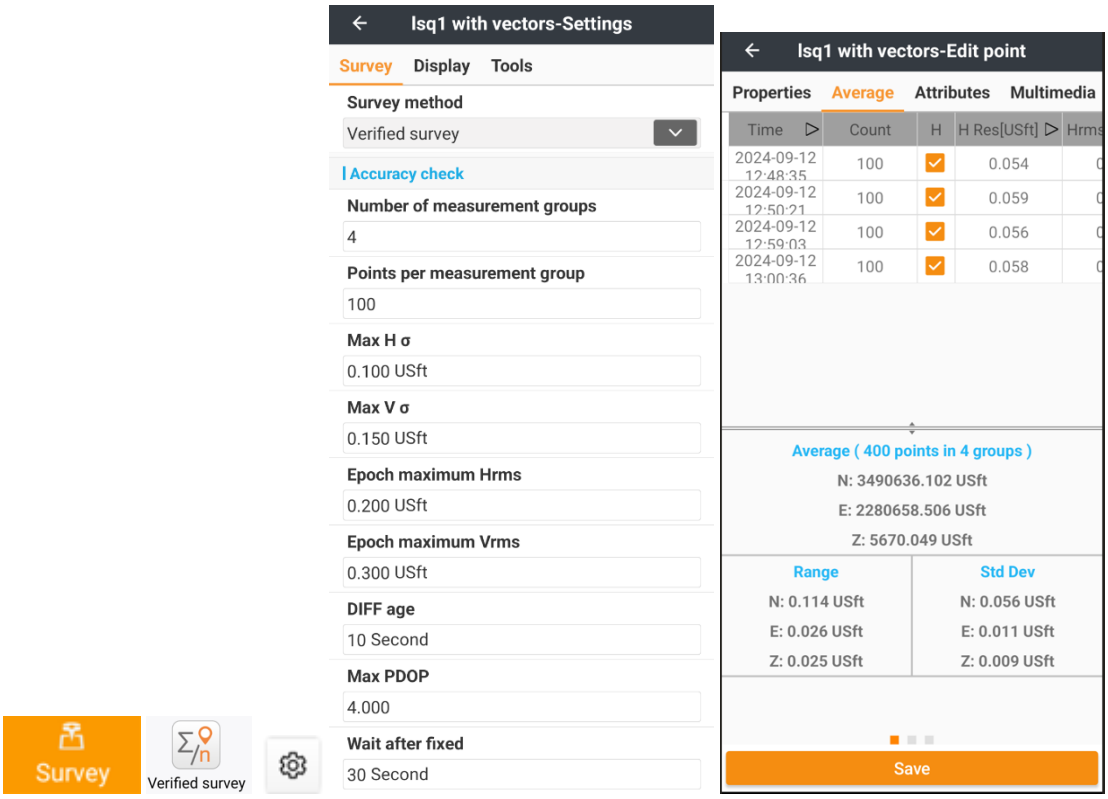
Verified survey: 

Point Average: 

2 or more (3 or more preferred) regular measurements using the same Point name

Since the Verified survey method forces a complete receiver initialization between groups, and allows for subsequently adding additional groups, Verified survey is the preferred measurement type for least squares analysis.

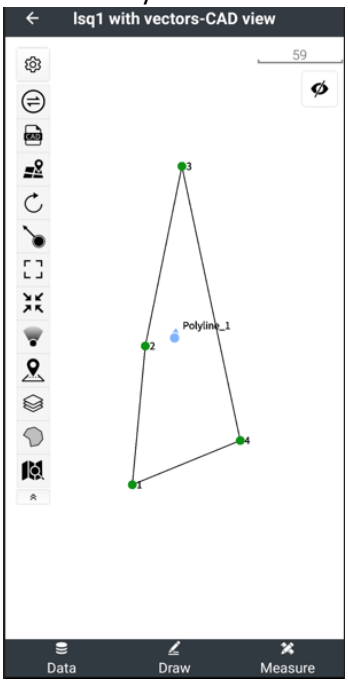
Start by storing each boundary corner as a unique point name. A verified survey with 4 measurement groups (3 redundancies), 100 points per group at 5Hz and a 30-second wait after receiver initialization is suggested:



After storing all the boundary corners, draw a closed polygon around the parcel. This will simplify the selection of parcel boundary points and their connections when computing the least squares analysis:

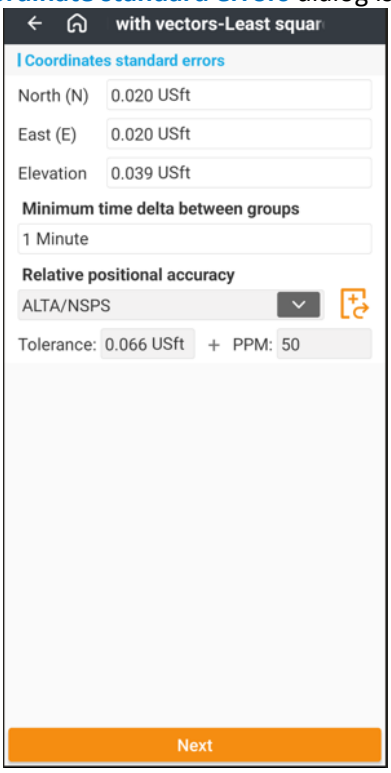
Survey > CAD View > Draw > Polyline, click around the parcel, close the final leg, Save

The parcel boundary will now have a closed polyline:



From the main menu **Tools** (tab) click on **Least squares**:

The **Coordinate standard errors** dialog is shown:



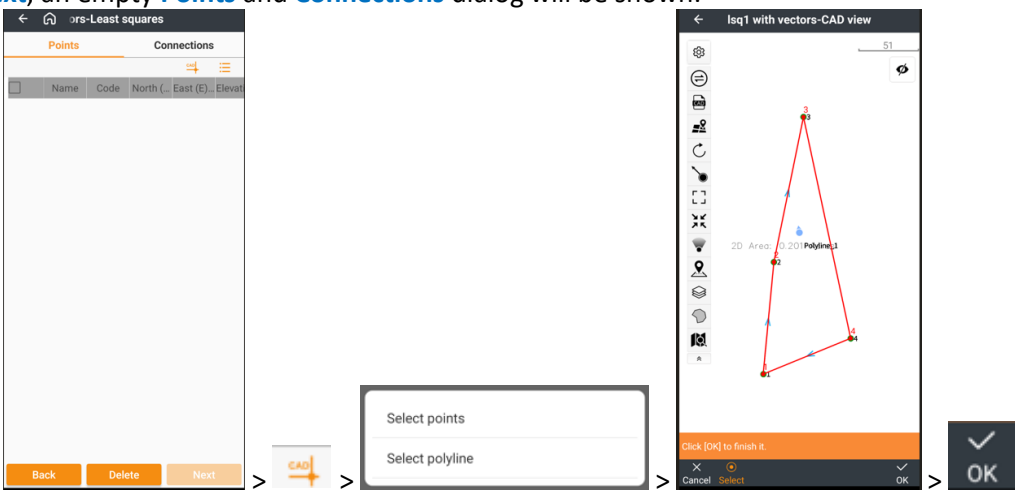
Measurement and setup errors obscure true coordinates for the Base(s). These standard errors estimate how repeated setups would be distributed around the true coordinates. They include centering errors, tribrach calibration errors, bubble errors, and pole runout for the Base setup. The values shown above are reasonable for a Base setup on a tripod – tribrach, a fixed height tripod would likely have a standard elevation error of only 0.02 feet.

Minimum time delta between groups will warn if GNSS measurements are taken very quickly, before the constellation has an opportunity to change. Because Verified surveys typically include a full GNSS engine reset between groups, and an additional 30-second delay after fix is included, 1-minute should be sufficient to insure independent groups.

The Tolerance is the maximum allowable error for a very short vector, PPM is multiplied by the vector length then added to the allowable error. For example, if the Tolerance is 0.066' + 50 ppm and the vector length is 480 feet:

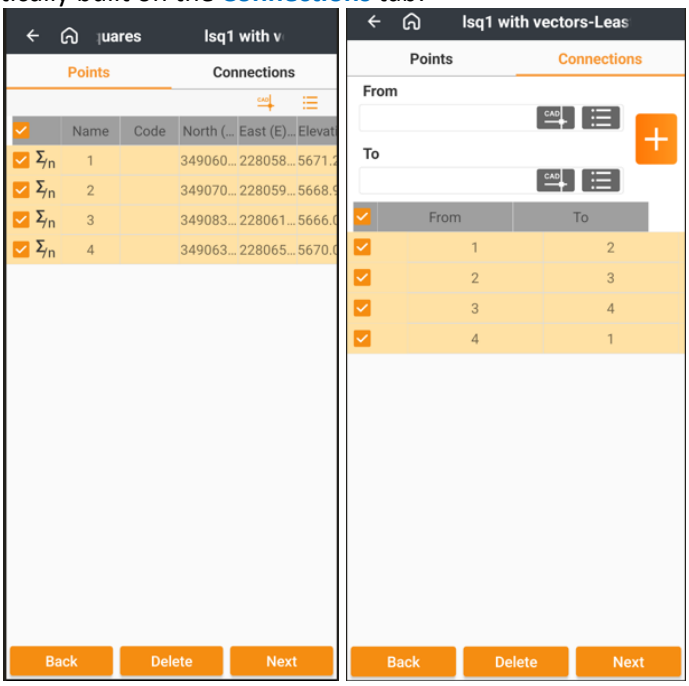
Allowable Error = 0.066 feet + 480 feet * 0.000050 (ppm) = 0.090 feet

Click **Next**, an empty **Points** and **Connections** dialog will be shown:

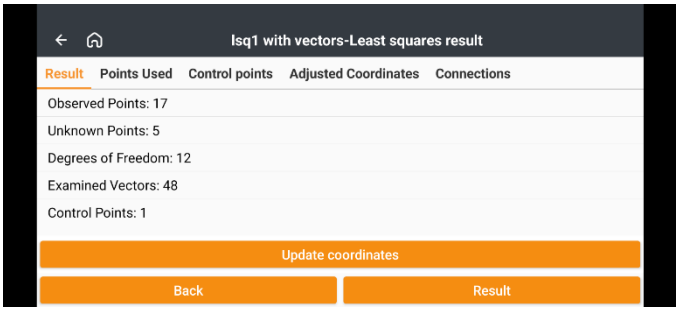


Click the **CAD** button, click **Select polyline**, click on the parcel boundary, then click **OK**.

The included boundary corners will be listed on the **Points** tab and the parcel vectors will be automatically built on the **Connections** tab:

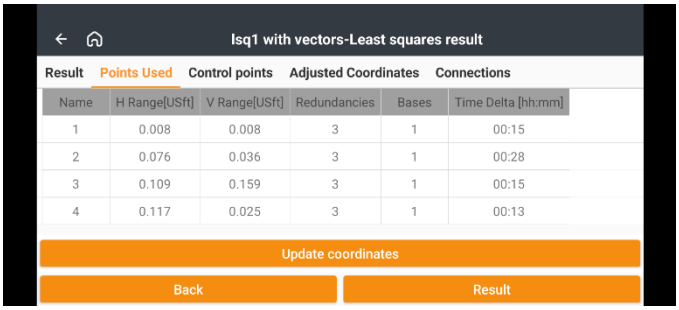


Click **Next**. LandStar will perform the least squares analysis and display the results:



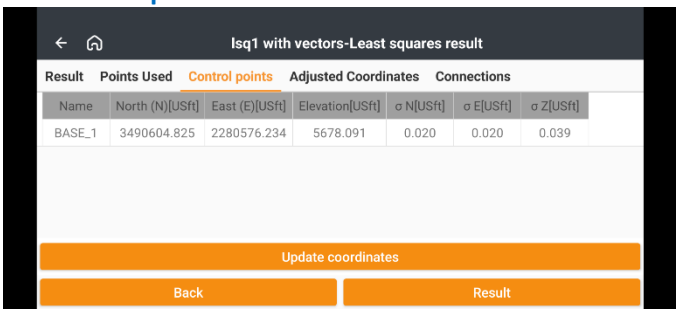
a summary of the analysis is shown.

Click on **Points Used** tab:



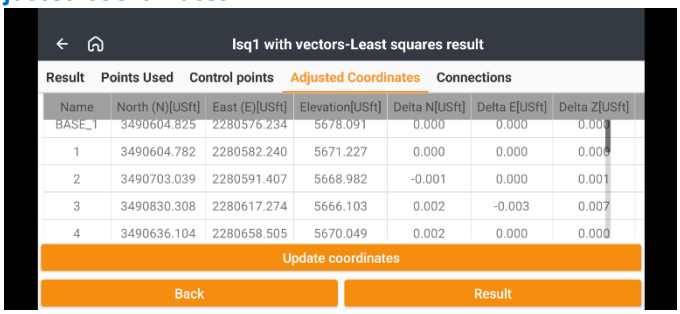
to view a summary of the unrestrained points and their group horizonal and vertical ranges. Note that if you make 4 group measurements, the redundancies will be 3, not 4.

Click on the **Control points** tab:



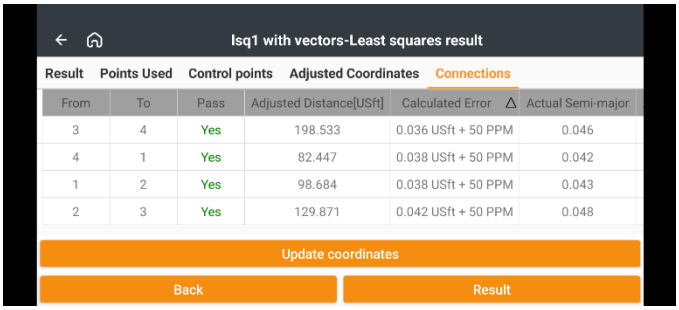
to view the Base **Control points** with the standard errors.

Click **Adjusted coordinates**:



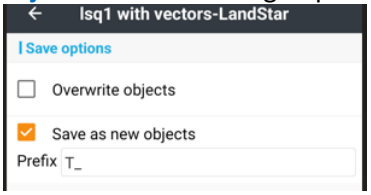
to view the computed adjustments to the group averages which best fit the measurement network. Usually, the suggested deltas will inconsequential when the underlying accuracy of GNSS measurements are considered.

Click the **Connections** tab:



to see which parcel vectors passed the tolerances, the best-fit adjusted vector lengths, the calculated measurement error in PPM excess, the actual semi-major error which represents the greatest horizontal error at a station, the allowable semi-major error computed by the vector length * PPM + tolerance and the ratio. Ratios less than 1 represent overly qualified vectors while ratios higher than 1 indicate a failing measurement.

If you want to save adjusted coordinates as new points, click on **Update coordinates** and select **Save as new objects** with a meaningful prefix:



> **Next**

It is also possible to overwrite the original measurements, but don't do this! Always make new measurements.

If you don't want letters in your point names, change the **Prefix** to a number like '900' to make new points in an entirely new point range.

Finally click on **Result** to write and optionally share a LSA Results PDF file:

Project Information	
Name	lsq1 with vectors
Created Time	2024-09-12 12:30:51
Operator	MES
Coordinate System	Utah (North) G18

Result	
Solution converged in 2 iterations	
Passed the Chi-Square test at the 95.00 significance level	
18.849 ≤ 34.979 ≤ 51.108	
Observed Points: 17	Unknown Points: 5
Degrees of Freedom: 12	Examined Vectors: 48
Control Points: 1	
Calculated RPA = 0.042 USft + 50 PPM	
Connections all pass.	
All confidence regions were computed using the following factors:	
1-D Expansion Factor : 1.96	
2-D Expansion Factor : 2.45	
Expansion factors for 95.00 confidence regions taken from normal distribution table	

Points Used

Point Name	Time Delta [hh:mm]	Redundancies	Bases
1	00:15	3	1
2	00:28	3	1
3	00:15	3	1
4	00:13	3	1

Control Points

Point Name	North (N) [USft]	East (E) [USft]	Elevation [USft]	σ N [USft]	σ E [USft]	σ Z [USft]
BASE_1	3490604.825	2280576.234	5678.091	0.020	0.020	0.039

Adjusted Coordinates

Point Name	North (N) [USft]	East (E) [USft]	Elevation [USft]	Delta N [USft]	Delta E [USft]	Delta Z [USft]	σ N [USft]	σ E [USft]	σ Z [USft]
BASE_1	3490604.825	2280576.234	5678.091	0.000	0.000	0.000	0.025	0.021	0.020
1	3490604.782	2280582.240	5671.227	0.000	0.000	0.000	0.029	0.026	0.026
2	3490703.039	2280591.407	5668.982	-0.001	0.000	0.001	0.029	0.033	0.028
3	3490830.308	2280617.274	5666.103	0.002	-0.003	0.007	0.031	0.035	0.030
4	3490636.104	2280658.505	5670.049	0.002	0.000	0.000	0.029	0.029	0.028

Connections

From	To	Pass	Adjusted Distance [USft]	Calculated Error	Actual Semi-major	Allow Semi-major	Ratio
3	4	✅	198.533	0.036 USft + 50 PPM	0.046	0.076	0.605
4	1	✅	82.447	0.038 USft + 50 PPM	0.042	0.070	0.594
1	2	✅	98.684	0.038 USft + 50 PPM	0.043	0.071	0.608
2	3	✅	129.871	0.042 USft + 50 PPM	0.048	0.072	0.664

Interpreting this report:

Passed the chi-square test at the 95% confidence interval means that the results of a chi-square test showed a statistically significant difference between observed and expected values, with a confidence level of 95%, indicating a strong likelihood that the observed result is not due to chance.

In this context, **passing** means that the calculated chi-square statistic was large enough to reject the null hypothesis at the 95% confidence level.

A 95% confidence interval means that if the test was repeated multiple times, 95% of the results would fall within the tolerance range.

The **Calculated RPA** is the worst-case connection:

Calculated RPA = 0.042 USft + 50 PPM

If the **Calculated RPA** for the worst-case connection is better than the PPM value alone, the **Calculated RPA** will be reported as 'Better than 50 PPM by -x.xxx'.

Calculated RPA = Better than 50 PPM by -0.281 USft

This often happens with long connections where the PPM allowance is significant:

Connections							
From	To	Pass	Adjusted Distance [USft]	Calculated Error	Actual Semi-major	Allow Semi-major	Ratio
14	12	✅	6649.465	Better than 50 PPM by -0.323 USft	0.010	0.398	0.025
11	14	✅	6640.921	Better than 50 PPM by -0.322 USft	0.010	0.398	0.025
12	13	✅	5878.363	Better than 50 PPM by -0.284 USft	0.010	0.360	0.027
13	11	✅	5808.160	Better than 50 PPM by -0.281 USft	0.010	0.356	0.028

In this case, the worst relative accuracy is connection 14 to 12. This vector is allowed to have:

6649.465 * 0.000050 + 0.066 = **0.398** feet allowable error

However, it statistically has a **0.010 foot** 95% uncertainty. The ratio is 0.025 which indicates that the measurement's accuracy is 40 times better than needed. In other words, the expected accuracy is better than 50 PPM by 0.281 feet.

This is good!

Notes

The **Least square** function is available in LandStar8 version 8.1.0.4.20240923 and higher.